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Generalized integral forms for friction, heat and mass transfer coefficients

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Abstract—The paper has the following objectives: (1) to draw attention to an improved approach of constructing analytical relationships for friction, heat and mass transfer coefficients based on the Reynolds flux concept and generalized substance transfer coefficient; (2) to show the way in which the suggested integral equation is used to derive relations for friction, heat and mass transfer coefficients; (3) to demonstrate the validity of the 'conformity principle' for limiting cases.

1. INTRODUCTION

Analysis of the requirements of safety and efficiency for modern high-power equipment calls for the development of relationships for predicting the hydrodynamics, heat and mass transfer characteristics of a coolant medium in a wide range of operating conditions and physical phenomena. In particular, it is important to take account of thermophysical properties, the contribution of inner sources (sinks), flows with injection (suction) and two-phase flows. As a rule, the analytical models of Petukhov [1], Novikov and Voskresenskiy [2], Kutateladze and Leontiev [3] and of Lyon [4] have been developed for solving certain specific problems and therefore are limited by concrete sets of assumptions that narrow the range of their application.

This paper attempts to solve the following problem, namely, to derive a general form of relationships for the distribution of the coolant medium parameters and also friction, heat and mass transfer coefficients proceeding from original formulations of conservation laws. The subsequent discussion and conclusions are confined to the conditions of inner problems for fully established velocity, temperature and concentration profiles in axisymmetrical channels.

2. GENERALIZED SUBSTANCE TRANSFER COEFFICIENTS

In accordance with definitions of friction factor and heat and mass transfer coefficients [1, 5], each of them is expressed in terms of the cross-section averaged weighted flow parameters (velocity, temperature, and concentration, see the first line in Table 1).

That the above definitions could be unified, it is expedient to use the concept of the Reynolds flux [5] (see the 2nd line in Table 1). Then the substance flux can be written as

$$J_{\mathrm{w}} = \omega Rm_{\mathrm{s}}(S_{\mathrm{w}} - \bar{S}). \tag{1}$$

The choice of the sign depends on specific flow conditions and is not shown here; it is considered to be positive by default. The factor ω appears for the Reynolds heat flux as the averaged specific heat of the flow when the heat transfer coefficient is defined on the basis of the Newton–Rikham law. When friction and mass transfer are considered, $\omega = 1$, $-\omega = 1$. Here, for the inner problems of hydraulics, heat and mass transfer in axisymmetric channels it is accepted that

$$\begin{aligned} S_{\mathrm{w}} - \bar{S} &= \langle (S_{\mathrm{w}} - S)\rho w^{\psi}\rangle / \langle \rho w^{\psi}\rangle \\ &= \int_0^1 (S_{\mathrm{w}} - S)\rho w^{\psi} R_{\ell}\,\mathrm{d}R \Big/ \int_0^1 \rho w^{\psi} R_{\ell}\,\mathrm{d}R \end{aligned} \tag{2}$$

where the following notation is used for the sake of brevity: $R_{\ell} = \ell R^{\ell-1}$ ($\ell = 1$ stands for a plane channel and $\ell = 2$ for a circular tube). The quantity S assumes the value of the enthalpy h (or temperature T) and concentration c when $\psi = 1$. The modulus $|S_{\mathrm{w}} - \bar{S}|$ is required for the axial velocity w (for $w_{\mathrm{w}} = 0$). Thus, in the absence of the slip on the wall, $w_{\mathrm{w}} = 0$, the weighted mean velocity is defined with the density as a weighing factor

$$\bar{w} = \langle \rho w\rangle / \langle \rho \rangle. \tag{3}$$

By definition the generalized substance transfer coefficient St_{s} and the axial mass flux $\langle \rho w\rangle$ are associated by the relationship

$$St_{\mathrm{s}} = Rm_{\mathrm{s}} / \langle \rho w\rangle. \tag{4}$$

Eliminating Rm_{s} from equations (1) and (4) results in the following dependence for the generalized substance transfer coefficient

$$\begin{aligned} St_{\mathrm{s}} &= J_{\mathrm{w}} / (\langle \rho w\rangle \omega (S_{\mathrm{w}} - \bar{S})) \\ &= J_{\mathrm{w}} \langle \rho w^{\psi}\rangle / (\langle \rho w\rangle \omega \langle (S_{\mathrm{w}} - S)\rho w^{\psi}\rangle). \end{aligned} \tag{5}$$

The physical meaning of St_{s} follows directly from

NOMENCLATURE

C_p	specific heat capacity
c	concentration, $(\varphi\rho)_g/\rho$
D	diffusion coefficient
g	gravitational acceleration
h	specific enthalpy
k	thermal diffusivity
$\mathscr{k}$	form factor
N	mass flux, $\{\rho w\}$
P	pressure
q	heat flux
R	relative coordinate, r/r_w
r	radius, distance from axis
S	generalized variable, $(w, h(T), c)$
T	temperature
t	time
$\mathbf{u}$	local velocity (vector)
$\mathbf{u}_{gf}$	phase difference velocity
v	radial velocity
w	axial velocity
z	axial coordinate.

Greek symbols

α	heat transfer coefficient
α_N	mass transfer coefficient
Γ	mass source
ν	kinematic viscosity
ξ	friction factor
ρ	density, $\{(\varphi\rho)_g+(\varphi\rho)_f\}$
τ	shear stress
φ	void fraction.

Subscripts, superscripts

D	diffusion
d	drift
g	gas {vapour}
f	fluid
s	substance
T	total
t	turbulent
w	wall
v	local sources.

Symbols

$*$	friction
$+$	friction scales
$\sim$	relative
$-$	weighted mean value
$\langle\,\rangle$	area average
$=$	tensor.

Dimensionless groups

Fr	Froude number
Pe	Peclet number
Pr	Prandtl number
Re	Reynolds number
Sc	Schimidt number
St	Stanton number.

equations (4) and (5) that define the generalized coefficient of substance transfer (momentum, heat and mass) as a measure of radial-to-axial substance transfer ratio.

2.1. *Profiles of flow parameters in the channel cross section*

In fully developed flows through axisymmetric channels of constant cross-section the main flows can vary both axially and radially. The simplest forms of other flows can be represented by axial and radial components of a turbulent flow under the conditions of the so-called linearly-varying flows [5]. Thus, to this type of problem belong different models of boundary-layer flows, as well as the 'slender' channel models of Lapin and Strelets [6]. Then, the changes in the mean value of the variable S under consideration (which is referred to mean axial velocity, enthalpy or concentration) are: (1) identical at each cross-section of the channel and (2) at least linear downstream.

To unify designations and reduce intermediate calculations when deriving relationships for the profiles of the parameters and friction, heat and mass transfer coefficients, it is convenient to use the concepts of generalized substance transfer coefficients [5] and the gradient transfer model. Mathematical resemblance between the three above-mentioned transfer processes (see the top line in Table 1) makes it possible to introduce a formally generalized equation, in which the substance flux J is expressed in terms of the transfer characteristic γ_T and the gradient of the transfer potential S as follows

$$J = \rho\gamma_T\,\partial S/\partial r \tag{6}$$

where $\gamma_T = \gamma+\gamma_S$ is the total (molecular (γ) + molar (γ_s)) substance transfer characteristic (coefficient), namely, the viscosity, thermal diffusivity or diffusion coefficient (see the third line in Table 1). Here emphasis should be placed on the fact that, according to Boussinesque's idea of putting turbulent friction formula in the form of Newton's laminar law, the turbulent heat and mass fluxes are reduced to the form which formally generalizes the Fourier and Fick laws. Scaling the variables in equation (6) with their values on the wall and integrating along the radius R yields the profile of variation for any of the potentials under consideration in the channel cross-section for the known substance flux and molecular–molar characteristics of transfer

$$S_w^+ - S^+ = Pe_{s*}\int_R^1 \frac{\tilde{J}}{\tilde{\rho}\tilde{\gamma}_T}\,dR. \tag{7}$$

Table 1. Formulations of substance transfer fluxes, numbers and coefficients

	Substance flux	Momentum flux	Heat flux	Mass flux
	$J = \rho\gamma_{\mathrm{T}}\dfrac{\partial S}{\partial r}$	$\tau = -\rho\nu_{\mathrm{T}}\dfrac{\partial w}{\partial r}$	$q = \rho k_{\mathrm{T}}\dfrac{\partial h}{\partial r}$	$N = \rho D_{\mathrm{T}}\dfrac{\partial c}{\partial r}$
1	Friction, heat or mass transfer coefficient	$\xi = \dfrac{8\tau_{\mathrm{w}}}{\langle\rho\rangle(\bar{w})^2} = \dfrac{8\tau_{\mathrm{w}}}{\langle\rho w\rangle\bar{w}}$	$\alpha = \dfrac{q_{\mathrm{w}}}{T_{\mathrm{w}}-\bar{T}}$; $St = \dfrac{\alpha}{\langle\rho w\rangle C_{\mathrm{p}}}$	$\alpha_N = \dfrac{N_{\mathrm{w}}}{(c-\bar{c})}$; $St_{\mathrm{D}} = \dfrac{\alpha_{\mathrm{N}}}{\langle\rho w\rangle}$
2	Reynolds flux $Rm_{\mathrm{s}} = J_{\mathrm{w}}/\omega(S_{\mathrm{w}}-\bar{S})$	for $w_{\mathrm{w}} = 0$ $Rm_\tau = \tau_{\mathrm{w}}/\bar{w}$	$Rm_q = q_{\mathrm{w}}/(h_{\mathrm{w}}-\bar{h})$	$Rm_N = N_{\mathrm{w}}/(c_{\mathrm{w}}-\bar{c})$
3	Transfer coefficient $\gamma_{\mathrm{T}} = \gamma+\gamma_{\mathrm{S}}$; $\left(\tilde{\gamma}_{\mathrm{T}} = \dfrac{\gamma_{\mathrm{T}}}{\gamma_{\mathrm{w}}}\right)$	$\tilde{\nu}_{\mathrm{T}} = \tilde{\nu}+\tilde{\nu}_{\mathrm{t}}$	$\tilde{k}_{\mathrm{T}} = \tilde{k}+\dfrac{Pr}{Pr_{\mathrm{t}}}\tilde{\nu}_{\mathrm{t}}$	$\tilde{D}_{\mathrm{T}} = \tilde{D}+\dfrac{Sc}{Sc_{\mathrm{t}}}\tilde{\nu}_{\mathrm{t}}$
4	Transfer number $Pe_{\mathrm{s}} = \langle\rho w\rangle r_{\mathrm{w}}/(\rho_{\mathrm{w}}\gamma_{\mathrm{w}})$ $Pe_{\mathrm{s}*} = w_* r_{\mathrm{w}}/\gamma_{\mathrm{w}}$	$Re = \langle\rho w\rangle r_{\mathrm{w}}/(\rho_{\mathrm{w}}\nu_{\mathrm{w}})$ $Re_* = w_* r_{\mathrm{w}}/\nu_{\mathrm{w}}$	$Pe = \langle\rho w\rangle r_{\mathrm{w}}/(\rho_{\mathrm{w}}k_{\mathrm{w}})$ $Pe_* = w_* r_{\mathrm{w}}/k_{\mathrm{w}}$	$Pe_D = \langle\rho w\rangle r_{\mathrm{w}}/(\rho_{\mathrm{w}}D_{\mathrm{w}})$ $Pe_{D*} = w_* r_{\mathrm{w}}/D_{\mathrm{w}}$
5	Substance of friction $S_* = J_{\mathrm{w}}/(\rho_{\mathrm{w}}w_*)$	$w_*^2 = \tau_{\mathrm{w}}/\rho_{\mathrm{w}}$	$h_* = q_{\mathrm{w}}/(\rho_{\mathrm{w}}w_*)$	$c_* = N_{\mathrm{w}}/(\rho_{\mathrm{w}}w_*)$
6	Variable S $S^+ = \dfrac{S}{S_*}$	for $w_{\mathrm{w}} = 0$; $w^+ = Re_*\displaystyle\int_R^1 \dfrac{\tilde{\tau}}{\tilde{\rho}\tilde{\nu}_{\mathrm{T}}}\mathrm{d}R$	$h_{\mathrm{w}}^+ - h^+ = Pe_*\displaystyle\int_R^1 \dfrac{\tilde{q}}{\tilde{\rho}\tilde{k}_{\mathrm{T}}}\mathrm{d}R$	$c_{\mathrm{w}}^+ - c^+ = Pe_{\mathrm{D}*}\displaystyle\int_R^1 \dfrac{\tilde{N}}{\tilde{\rho}\tilde{D}_{\mathrm{T}}}\mathrm{d}R$
7	Stanton number $(St_{\mathrm{s}}\omega Pe_{\mathrm{s}})^{-1} =$ $\displaystyle\int_0^1 \dfrac{\rho w^\psi}{\langle\rho w^\psi\rangle}\left(\int_R^1 \dfrac{\tilde{J}}{\tilde{\rho}\tilde{\gamma}_{\mathrm{T}}}\mathrm{d}R\right)R_\ell\,\mathrm{d}R$	for $w_{\mathrm{w}} = 0, \psi = 0, \omega = 1$; $8/(\xi Re) =$ $\displaystyle\int_0^1 \dfrac{\rho}{\langle\rho\rangle}\left(\int_R^1 \dfrac{\tilde{\tau}}{\tilde{\rho}\tilde{\nu}_{\mathrm{T}}}\mathrm{d}R\right)R_\ell\,\mathrm{d}R$	for $\psi = 1, \omega = C_{\mathrm{pw}}/\bar{C}_{\mathrm{p}}$; $(St\omega Pe)^{-1} =$ $\displaystyle\int_0^1 \dfrac{\rho w}{\langle\rho w\rangle}\left(\int_R^1 \dfrac{\tilde{q}}{\tilde{\rho}\tilde{k}_{\mathrm{T}}}\mathrm{d}R\right)R_\ell\,\mathrm{d}R$	for $\psi = 1, \omega = 1$; $(St_{\mathrm{D}}Pe_{\mathrm{D}})^{-1} =$ $\displaystyle\int_0^1 \dfrac{\rho w}{\langle\rho w\rangle}\left(\int_R^1 \dfrac{\tilde{N}}{\tilde{\rho}\tilde{D}_{\mathrm{T}}}\mathrm{d}R\right)R_\ell\,\mathrm{d}R$

Using the definitions given in Table 1 one can easily reconstruct specific relationships for the profiles of axial velocity, enthalpy (temperature) and concentrations (see the sixth line) from integral (7). They prove to be a generalization of the relationships obtained by Petukhov and Popov [1] for velocity and temperature profiles that are not limited by the assumption about a linear distribution of viscous stresses.

2.2. *Basic integral relationship for generalized substance flux*

As a result of scaling the variables in equation (5) one obtains

$$St_{\mathrm{s}}^{-1} = \langle\tilde{\rho}w^+\rangle\omega(S^+ - \bar{S}^+) \tag{8}$$

where

$$\langle\tilde{\rho}w^+\rangle = \frac{\langle\rho w\rangle}{\rho_{\mathrm{w}}w_*} = \frac{Pe_{\mathrm{s}}}{Pe_{\mathrm{s}*}},$$

$$\omega = \begin{cases} 1\text{—friction and mass transfer} \\ C_{\mathrm{PW}}/\bar{C}_{\mathrm{p}}\text{—heat transfer} \end{cases}$$

$$\bar{C}_{\mathrm{p}} = (h_{\mathrm{w}}-\bar{h})/(T_{\mathrm{w}}-\bar{T}) = \int_T^{T_{\mathrm{w}}} C_{\mathrm{p}}\,\mathrm{d}T/(T_{\mathrm{w}}-\bar{T}). \tag{9}$$

Inserting the integral relationship for the substance profile (7) with allowance for equation (2) or (3) into relationship (8) results in the desired integral form for the generalized substance transfer coefficient

$$\frac{1}{St_{\mathrm{s}}} = Pe_{\mathrm{s}}\omega\int_0^1 \frac{\rho w^\psi}{\langle\rho w^\psi\rangle}\left(\int_R^1 \frac{\tilde{J}}{\tilde{\rho}\tilde{\gamma}_{\mathrm{T}}}\mathrm{d}R\right)R_\ell\,\mathrm{d}R. \tag{10}$$

Thus, St_{s} is expressed is terms of the profile of the substance flux J being transferred in the radial direction as well as in terms of the local physical properties ω, ρ and total (molecular+molar) transfer characteristic. It should be stressed that equation (10) is universal, that is, the insertion of appropriate fluxes and substance characteristics transforms it into the friction, heat and mass transfer coefficients. This can be easily verified using definitions given in Table 1 (see the last line).

3. SUBSTANCE FLUX DISTRIBUTION

The method developed can be extended to two-phase flows, so it is convenient to use conservation laws in a form equally acceptable for both single-phase and two-phase flows. For this purpose, it appears expedient to discribe the two-phase flow on the basis of the drift flux model [7]. Table 2 shows the drift flux

Table 2. Non-conservative (transportable) forms of conservation law equations, two-phase flow description is based on drift flux model [7]

Field equations of:	Definitions of parameters
Mixture continuity $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$	$\rho = (\varphi\rho)_g + (\varphi\rho)_f$; $\rho\mathbf{u} = (\varphi\rho\mathbf{u})_g + (\varphi\rho\mathbf{u})_f$; where $\varphi_g = 1 - \varphi_f$
Convective diffusion $\rho\frac{\partial c}{\partial t} + \rho\mathbf{u} \cdot \nabla c = -\nabla \cdot (\mathbf{N}_T) + \Gamma$	$c = (\varphi\rho)_g/\rho$; $\mathbf{N}_T = \mathbf{N}_t + \mathbf{N}_d$; $\mathbf{N}_d = c(\varphi\rho)_g\mathbf{u}_{gf}$
Mixture energy $\rho\frac{\partial h}{\partial t} + \rho\mathbf{u} \cdot \nabla h = -\nabla \cdot (\mathbf{q}_T) + q_v$	$h = [(\varphi\rho h)_g + (\varphi\rho h)_f]/\rho$; $\mathbf{q}_T = \mathbf{q}_t + \mathbf{q}_d$; $\mathbf{q}_d = c\rho_f\mathbf{u}_{gf}(h_g - h_f)$
Mixture momentum $\rho\frac{\partial \mathbf{u}}{\partial t} + \rho\mathbf{u} \cdot \nabla\mathbf{u} = \nabla \cdot (\bar{\bar{\tau}}_T) - \nabla P + \rho\mathbf{g}_T$	$\mathbf{u} = [(\varphi\rho\mathbf{u})_g + (\varphi\rho\mathbf{u})_f]/\rho$; $\bar{\bar{\tau}}_T = \bar{\bar{\tau}}_t + \bar{\bar{\tau}}_d$; $\bar{\bar{\tau}}_d = \frac{c}{1-\varphi}\rho_f\mathbf{u}_{gf}\mathbf{u}_{gf}$; $\rho\mathbf{g}_T = \rho\left(\mathbf{g} + \frac{\Gamma}{\rho}\mathbf{u}_{gf}\right)$

model modified for the purposes of the present paper. The law of propagation of a light phase is described by means of the convective diffusion equation. Equations of conservation laws are used in a non-conservative form. Here the density ρ, mass velocity $\rho\mathbf{u}$ and enthalpy h represent their two-phase characteristics defined on the right-hand side of Table 2. The terms with the subscript d taking account of the light phase drift are added to the corresponding turbulent substance fluxes with the subscript t. Additional acceleration due to vapour formation at the phase interface can be written together with the hydrostatic component. Thus, a form is obtained that is identical for both single-phase and two-phase flows.

Furthermore, the analysis will be confined to steady-state flow conditions for an axisymmetrical flow in the $(r$–$z)$ geometry. Using the boundary-layer approximation [6] it is possible to write the conservation law equations presented in Table 2 in a generalized and unified form by means of the single substance transfer equation

$$\frac{1}{r^{k-1}}\frac{\partial}{\partial r}(r^{k-1}J) = \rho w\frac{\partial S}{\partial z} + \rho v\frac{\partial S}{\partial r} - I_V \qquad (11)$$

where w is the axial velocity and v is the radial velocity. In the present paper the source term I_V is coupled with the pressure gradient and hydrostatic component in the motion equation, whereas in the energy and mass transfer equation it represents the heat and mass source (sink).

Scaling the variables in equation (11) gives

$$\frac{1}{R^{k-1}}\frac{\partial}{\partial R}(R^{k-1}\tilde{J}) = \tilde{\rho}w^+\frac{\partial S^+}{\partial \tilde{z}} + \tilde{\rho}v^+ + \frac{\partial S^+}{\partial R} - \tilde{I}_V \qquad (12)$$

where $\tilde{I}_V = I_V r_w/J_w$, $\tilde{z} = z/r_w$.

The integration of the latter equation first with the variable upper limit R and then up to the wall, $R = 1$, and the combination of the integrals obtained yield an equation for the local substance flux

$$\tilde{J} = R\left[1 - \frac{1}{k}\left(\langle Sz\rangle - \frac{\langle Sz\rangle_R}{R^k} + \langle Sr\rangle - \frac{\langle Sr\rangle_R}{R^k} - \langle\tilde{I}_V\rangle + \frac{\langle\tilde{I}_V\rangle_R}{R^k}\right)\right] \qquad (13)$$

here the following designations are used for the sake of brevity

$$\langle Sz\rangle = \int_0^1 \tilde{\rho}w^+\frac{\partial S^+}{\partial \tilde{z}}R_k\,dR \qquad (14)$$

$$\langle Sr\rangle = \int_0^1 \tilde{\rho}v^+\frac{\partial S^+}{\partial R}R_k\,dR \qquad (15)$$

$$\langle Sz\rangle_R = \int_0^R \tilde{\rho}w^+\frac{\partial S^+}{\partial \tilde{z}}R_k\,dR \qquad (16)$$

$$\langle Sr\rangle_R = \int_0^R \tilde{\rho}v^+\frac{\partial S^+}{\partial R}R_k\,dR \qquad (17)$$

$$\langle\tilde{I}_V\rangle = \int_0^1 \tilde{I}_V R_k\,dR \qquad (18)$$

$$\langle\tilde{I}_V\rangle_R = \int_0^R \tilde{I}_V R_k\,dR. \qquad (19)$$

Relationship (13), describing radial distribution of the substance flux being transferred, and equations (7) and (10) enable one to calculate radial profiles and generalized substance transfer coefficients. As is seen from equation (13), the contribution of each of the effects under consideration (convective axial and radial transfer, sources (sinks), etc.) to the substance flux being transferred can be described as a correction for linear distribution.

4. ANALYTICAL RELATIONSHIPS FOR FRICTION FACTOR, HEAT AND MASS TRANSFER COEFFICIENTS

Assuming that the axial pressure gradient does not change in the channel cross-section yields the following relationship for the friction factor from equations (13) and (10)

$$\frac{8}{\xi} = Re\int_0^1 \frac{\rho}{\langle\rho\rangle}\left(\int_R^1 \frac{R}{\tilde{\rho}\tilde{v}_{\mathrm{T}}}\right.$$
$$\times\left\{1-\frac{1}{\ell}\left[\langle Vz\rangle-\frac{\langle Vz\rangle_{\mathrm{R}}}{R^{\ell}}+\langle Vr\rangle-\frac{\langle Vr\rangle_{\mathrm{R}}}{R^{\ell}}\right.\right.$$
$$\left.\left.\left.-\frac{1}{Fr_*}\left(\langle\tilde{\rho}\rangle-\frac{\langle\tilde{\rho}\rangle_{\mathrm{R}}}{R^{\ell}}\right)\right]\right\}\mathrm{d}R\right)R_{\ell}\,\mathrm{d}R \quad (20)$$

where the integrals $\langle Vz\rangle$, $\langle Vr\rangle$, $\langle Vz\rangle_{\mathrm{R}}$ and $\langle Vr\rangle_{\mathrm{R}}$ are identical to relationships (14–17) when substituting w for variable S and

$$\langle\tilde{\rho}\rangle = \int_0^1 \tilde{\rho}R_{\ell}\,\mathrm{d}R \quad (21)$$

$$\langle\tilde{\rho}\rangle_{\mathrm{R}} = \int_0^R \tilde{\rho}R_{\ell}\,\mathrm{d}R \quad (22)$$

$$Fr_* = \frac{\tau_{\mathrm{w}}}{\rho_{\mathrm{w}}gr_{\mathrm{w}}}. \quad (23)$$

It is true to say that equation (20) generalizes the integral obtained by Petukhov and Popov [1] for the friction factor not only with respect to the functional form of accounting for the profile density but it also makes allowance for the axial $\langle Vz\rangle$ and radial $\langle Vr\rangle$ flow accelerations thus generalizing the two-phase flow model of Sato *et al* [8].

Under the assumption that the axial enthalpy (and concentration) gradients are not functions of the radial coordinate, it is possible to obtain the following expression from equations (13) and (10) after integration by parts

$$\frac{1}{Nu} = \omega\int_0^1\left[\left(1-\frac{\langle Hr\rangle}{\ell}+\frac{\langle\tilde{q}_{\mathrm{V}}\rangle}{\ell}\right)\left(\int_0^R\frac{\tilde{\rho}\mathrm{w}^+\mathrm{R}_{\ell}\,\mathrm{d}R}{\langle\tilde{\rho}w^+\rangle}\right)^2\right.$$
$$\left.+\left(\frac{\langle Hr\rangle_{\mathrm{R}}}{\ell}-\frac{\langle\tilde{q}_{\mathrm{V}}\rangle_{\mathrm{R}}}{\ell}\right)\int_0^R\frac{\tilde{\rho}w^+R_{\ell}\,\mathrm{d}R}{\langle\tilde{\rho}w^+\rangle}\right]\frac{\mathrm{d}R}{\tilde{\rho}k_{\mathrm{T}}R_{\ell}} \quad (24)$$

where the integrals $\langle Hr\rangle$ and $\langle Hr\rangle_R$ are identical to relationships (15), (17) and

$$\langle\tilde{q}_{\mathrm{V}}\rangle = \int_0^1 \tilde{q}_{\mathrm{V}}R_{\ell}\,\mathrm{d}R \quad (25)$$

$$\langle\tilde{q}_{\mathrm{V}}\rangle_R = \int_0^R \tilde{q}_{\mathrm{V}}R_{\ell}\,\mathrm{d}R. \quad (26)$$

The integral for the mass transfer coefficient $(St)_{\mathrm{D}}$ is functionally identical to equation (24), differing from it by the factor $\omega = 1$.

Comparison of equation (24) with the relationships found in references [1–4] allows a conclusion that this equation is a generalization of the Lyon integral for flows with allowance for injection (suction) and inner heat sources (sinks). This is confirmed by the validity of the 'conformity principle', in limiting cases inclusive, namely: (1) when $v \to 0$, the effect of radial transfer is neglected and the Novikov–Voskresenskiy type of the relationship [2] is derived which describes heat transfer for a flow with inner heat sources; (2) when $v \to 0$ and $q_{\mathrm{v}} \to 0$, the Petukhov–Popov relationships [1] are obtained; (3) when $v \to 0$ and $q_{\mathrm{v}} \to 0$ and physical properties are constant, the Lyon classical integral [4] is derived.

It should be stressed that the general integral forms (20) and (24) can be reduced to a number of other correlations found in the literature, though the mechanism of such a reduction is beyond the scope of the present paper. Thus, in particular, it can be shown that under the appropriate set of acceptable assumptions, for example, with respect to the injection (suction) models and γ_{T}, the correlations of Kutateladze–Leontiev [3] are obtained for the 'limiting laws' of friction, heat and mass transfer. Due to the assumption that the cross-sectional distribution of void fraction and other two-phase flow parameters is piecewise uniform, integral (20) amounts to a 'two-zone' model [9] and demonstrates that the problem can be solved in quadratures.

5. CONCLUSION

The paper presents a simple approach to the construction of generalized integral relationships for the friction factor, and heat and mass transfer coefficients by using the Reynolds flux concept and the generalized substance transfer coefficient within the boundary-layer assumptions. The contribution of momentum, heat and mass transfer as well as their sources and sinks in the channel cross-section are taken into account.

Unlike the well-known Kutateladze–Leontiev relationships [3] for the 'limiting laws' of friction, heat and mass transfer and the Petukhov–Popov relationships [1], the integral forms derived have a greater capacity for generalization, and the effects under study can be expressed in an additive form. It seems to be of considerable importance for criterion evaluation of the contribution of the effects involved. Besides, it is not the absolute value of the effect that matters, but the difference between the area-weighted value and its integral mean value.

The general integral relationships worked out are recommended as a basis for the development of new phenomenological models of friction, heat and mass transfer coefficients.

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